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OBJECT TRACKING AND DYNAMIC ESTIMATION ON EVIDENTIAL GRIDS

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ABSTRACT

Autonomous driving is one of the most challenging tasks of the automotive industry. As a subtask, the estimation of driveable and non driveable space is often solved by applying occupancy grids. The information about non driveable space can be used to improve object tracking. This paper presents an approach for object tracking and modelling in an occupancy grid map. Tracking objects on grid cells yields the advantage of a consistent environmental model on the occupancy grid map. We introduce the occupancy grid map as the only information source for the object tracking module. Taking advantage of the Dempster Shafer theory, a dynamic belief of conflicting cells can be estimated. This dynamic belief is then accumulated in a tracked object model. This is a grid based free form object model that uses an object local grid map representation to model vehicles in urban environment. We reduce false positives and initialization time by maintaining a dynamic belief for each object.

1. INTRODUCTION

An accurate environmental perception is a requirement for advanced driver assistant systems (ADAS) and mandatory for autonomous vehicles. As a modeling technique of the static environment, the occupancy grid framework proposed in [1] has dominated the scope and has been applied successfully to multiple systems [2] [3]. The original version of occupancy grid mapping suffers from artefacts produced by dynamic objects [4]. To prevent these issues, there are several techniques to filter out dynamic information or use them to set up grid map based tracking approaches. The basic idea of occupancy grid mapping is to model the environment as a set of discrete cells containing probabilities of the presence (occupancy) or absence (freeness) of an object. Elfes [1] proposes using a binary bayesian filter modeling the probability that a cell is occupied. In recent literature this idea has been extended by the Dempster Shafer theory (DST) using independent belief masses for occupancy and freeness. This enables the resolution of conflicts between independent contradictory measurements. Moras [5] presents an approach using an evidential dynamic detection based on the measured conflict of a cell. We extend this dynamic detection with an estimation that a cell is static using neighbouring cells.
Representing a consistent environment model on grid maps including dynamic objects turns out to be a challenge. Modelling object dynamics in grid maps is described in several publications [5] [3] [2]. However there are still open questions how to use the extended information generated by a grid map in the object tracking later on.

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Two basic techniques exist for modelling dynamics in occupancy grid maps.

- Cell attribute extension: extend each grid cell with dynamic information and track each of these cells individually [6].
- Associate cells to Tracks: associate a grid cell or cluster to a separated filter and keep track of the moving cells independently such as proposed in [3] [2].

Using the cell association technique for object tracking, information can be gathered in two ways [7]:

- Extract features describing the tracked object from an occupancy grid map and transform these feature to vehicle coordinates. A feature can be the shape or contour of an object. The tracking uses the extracted features to update the object states.
- Use the cell representation of the occupancy grid map directly in object tracking. The object state is represented in grid coordinates and is updated by a set of cells.

In this paper we propose a novel approach using grid cells extended with dynamic belief masses to set up and validate new object tracks. Using theses grid cells we associate multiple cells to a moving object employing the cell representation directly to update the object state.
We set up particle filtered objects that accumulate dynamic measurements in a separated Dempster Shafer belief mass. These accumulated belief masses will determine if the target is generated from clutter or from a real moving object (false positive reduction). In some highway scenarios, the tracking suffers from uncertain distance measurements of the road boundary. In such scenarios we use dynamic beliefs for pruning ghost objects before they could produce failures in driver assistant systems. We show that dynamic cell tracking could be used in urban and highway scenarios (see section 4).
We use a particle filter to track a dynamic cell cluster and then conceptually detach these cells and put them into an object local grid. Each particle consist of a cluster of grid cells. Therefore a particle set allows free form modelling on multiple clusters. These clusters are accumulated in an object local grid which represents the shape of an object as a $footprint$.
This paper is structured as follows. Section 2 gives a brief overview about our occupancy grid mapping and feature extraction. In section 3 we present our particle filter based tracking and give an overview of the track management. Some experimental results are given in section 4 and finally in section 5 conclusions and future work are discussed.

2. OCCUPANCY GRID MAPPING AND DYNAMIC ESTIMATION

In our case a complete representation of the environment is built from the measurements of multiple multilayer lidar sensors. We accumulate these measurements using a grid based fusion technique in a single occupancy grid map using DST. At the end of this section we introduce an approach for grid based dynamic estimation based on the work of [5].

2.1. Dempster Shafer Theory on Occupancy Grid Maps

The Dempster Shafer theory is a mathematical theory allowing the combinations of evidences [8]. An evidence can be estimated by multiple sources and an own degree of belief. In DST all possible hypothesis of a system are defined by a set Ω of mutually exclusive propositions. We define for occupancy grid mapping the set as $\Omega = \{f, o\}$ where f is the state for a free and o is the state for an occupied cell. The elements of the powerset $2^\Omega = \{\emptyset, \{f\}, \{o\}, \{f, o\}\}$ can be used to represent the actual state of the system. DST uses mass functions $m : 2^\Omega \mapsto [0, 1]$ assigning a belief mass to each element of the power set. Notice that the sum over all mass function is defined by: $\sum_{A \in 2^\Omega} m(A) = 1$.

$$m_{M \oplus N}(A) = \frac{1}{\eta} \sum_{B \cap C = A \neq \emptyset} m_M(B) \cdot m_N(C) \tag{1}$$
$$\eta = 1 - \sum_{B \cap C = \emptyset} m_M(B) \cdot m_N(C)$$

The set $A \subseteq \Omega$ contains all elements that represent a state of interest. Dempster's rule for combination (see equation 1) is used to combine the two sets of masses m_M and m_N. Where η can be regarded as the agreement of the belief of the masses. The fusion process of an occupancy grid map is committed on cell level.
Extending the idea of occupancy grid mapping, we create separated measurement grid layers for each purpose. $Layer_1$ encompasses the original idea of grid mapping and builds a model for the static environment in the view of the ego vehicle. In $Layer_1$ we use the power set 2^Ω. The measurement grid map and the occupancy grid map itself are both grid maps with the same attributes. So we are defining two mass functions: $m_{M,k}()$ as the mass function for the occupancy grid and $m_{S,k}()$ for the measurement grid.

2.2. Fusion Architecture

Accumulating information gathered from multiple sensors at different time slots a fusion process is needed in order to build a consistent grid map. For each laser scanner an own measurement grid map ($MeasGrid$) is built (see figure 1). So the process of occupancy grid mapping is triggered by every lidar scan. All measurement grids have to be in the same coordinates as the current grid map ($GridMap_k$) representation. Therefore each measurement has to be mapped to a joint coordinate system. Matching the measurement grid with the occupancy grid ($GridMap_{k-1}$) the accurate vehicle position and orientation $VehGridPos$ within the grid map has to be estimated. Using the estimated $VehGridPos$ each measurement grid is fused with the occupancy grid map applying equation 1. The applied sensor model is split into two processes: the free space and the occupied space estimation. This separation is done because the free space estimation require a lot more sensor specific tuning. As long as similar laser scanners with the same

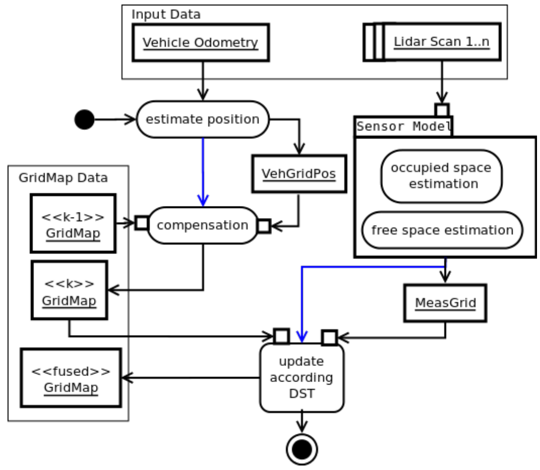

Figure 1: System Fusion Architecture in a UML 2.0 activity diagram. Blue arrows describe activity transitions. Black arrows show the flow of objects.

update frequency are used, a measurement grid map can be fused to the occupancy grid map using Dempster's rule of combination shown in equations 2 . . . 4.

$$m_{M,k}(O) = m_{M,k-1 \oplus S,k}(O) \tag{2}$$
$$m_{M,k}(F) = m_{M,k-1 \oplus S,k}(F) \tag{3}$$
$$m_{M,k}(\Omega) = 1 - m_{M,k}(O) - m_{M,k}(F) \tag{4}$$

The mass $m_{S,k}$ is measured by an individual scanner S at time k ($F = \{f\}$,$O = \{o\}$).

2.3. Dynamic Estimation

A grid based dynamic detection is based on the assumption that a laser beam aimed at a static object should always end in the same cell. This assumption holds when an accurate vehicle odometry is available for map compensation. Using the occupancy grid map as a reference for all measurements accumulated until time slot k the current measurement can be matched against the map to enable two assumptions of the dynamic state: On the one hand a laser beam that ends in an occupied cell is an evidence of a static measurement. On the other hand a beam that ends in a free cell is an evidence of dynamic measurement (see equation 5).
The dynamic detection in occupancy grid maps is extensively studied with respects to the used grid mapping algorithm. Both [9] and [3] use conflicting measurements in bayesian occupancy grids. In contrast, [5] uses the same assumption of conflicting measurements in evidential grids. To validate the assumption of a dynamic measurement, it is useful to generate an estimation that a cell is not occupied by a dynamic object. These results can be integrated in a Dempster Shafer belief mass. For the dynamic estimation we define a second power set $2^\Delta = \{\emptyset, \{d\}, \{\overline{d}\}, \{d, \overline{d}\}\}$, where d is the state of a dynamic measurement and $\overline{d}$ is the state of a contradictory measurement. Representing dynamic measurements of the environment, we create a separate grid map layer $Layer_2$ that includes all dynamic measurement. The belief masses $m_{S,k}(D)$ and $m_{S,k}(\overline{D})$ describe a measurement of the dynamic and the not dynamic state ($D = \{d\}, \overline{D} = \{\overline{d}\}$). For the term $m_{S,k}(D)$ the

motion detection algorithm proposed in [5] is used here. Equations 5 and 6 show the measurement of a conflict estimated to be dynamic as proposed in [5].

$$C_1 = m_{S,k}(O) * m_{M,k-1}(F), \text{ when } f \text{ turns to } o \quad (5)$$
$$C_2 = m_{S,k}(F) * m_{M,k-1}(O), \text{ when } o \text{ turns to } f \quad (6)$$

Where C_1 describes a cell which is currently occupied with a dynamic object (new dynamic). C_2 describes a cell which is freed by an moving object (old dynamic). Using the conflict C_1 between the measurement and the map, we propose to extend this motion detection algorithm with an estimation $\overline{d}$, which determines that the current cell is not dynamic. Every measurement that falls on a free cell could be estimated as dynamic but due to discretization errors and noisy measurements we need an estimation that the cell is not dynamic $\overline{D}$. For every cell we take the state of neighbouring cells into account:

$$m_{S,k}(D) = C_1 \quad (7)$$
$$m_{S,k}(\overline{D}) = \alpha \sum_{n \in N} m_{M_n,k-1}(O) \quad (8)$$

The mass belief function for $m_{S,k}(\overline{D})$ can be described as the sum of all neighbour cells ($n \in N$) where α is a normalization constant and N is the set of relevant neighbours. We propose to take the belief of all eight neighbouring cells as the noise measurement sum for belief mass $m_{S,k}(\overline{D})$. For the belief mass of $m_{S,k}(D)$ we only take the conflict measurement C_1 into account, because C_1 describes where the moving object is currently located.
Before we can combine the belief masses of $Layer_2$, the movements of all object local grid maps have to be predicted. Therefore this update is separated from the static environment beliefs and is discussed in section 3.4.

3. DYNAMIC CLUSTER TRACKING

At this stage we have introduced a layered occupancy grid map built from several sensors. $Layer_1$ for the static environment determine a cell is occupied or free and $Layer_2$ for the dynamic environment estimating the belief for a dynamic and a not dynamic cell. In this chapter we only refer to $Layer_2$ in order to track objects from the dynamic estimation of the measurement grid map.
To generate features from the dynamic estimation we build clusters for all dynamic cells using the $db-scan$ algorithm [10]. Each cluster consists of a minimum of two cells each containing two beliefs: one for the dynamic and one for the not dynamic belief. Once a dynamic belief for each cell cluster is estimated, it can be used to set up object tracks. In contrast to other tracking approaches dynamic cell clusters, allow us to track the dynamic environment only.
The object model of a single tracked object is shown in figure 2.
The track management, object model, association strategy and tracking algorithm are described in this section.

3.1. Particle Filters

A dynamic model and tracking algorithm is required for estimating directions and velocities. Cell clusters are a viable starting point. Vu [2] tracks dynamic clustered cells by using a Kalman filter and interactive multi model (IMM) algorithm. In contrast, [11] and [12] use particle filters. Particle filters belong to the monte carlo methods. They approximate the posterior density of the state space with a known transition model combined with a noise assumption. Or in other words particle filter approximate inference in partially observable Markov chains. Notice the state of the Markov chain at time t is given by x_t. Furthermore, the state x_t depends on the previous state x_{t-1} according to the probabilistic law $p(x_t|u_t, x_{t-1})$, where u_t is the control arrived in the intervall $[t-1, t]$. The state in the Markov chain is not observable directly. But we can use the measurement z_t, which can be used as a probabilistic projection to the true state x_t via the propabilistic law $p(z_t|x_t)$. For the object tracking case, $p(x_t|u_t, x_{t-1})$ is referred as a motion model, and $p(z_t|x_t)$ as the measurement model (see equation 9). Applying a particle filter to the object tracking problem the controls (u_t) are usually velocity and yaw-rate of the tracked object. The state of an object tracked with particle filters is represented as multiple hypothesis (i.e. particles). Each particle contains a state hypothesis (x_t^p) of the tracked object and its weight (w_t^p). As a first step of the particle filter algorithm, the state of a particle is predicted using a motion model (line 3 of algorithm 1). The motion model is

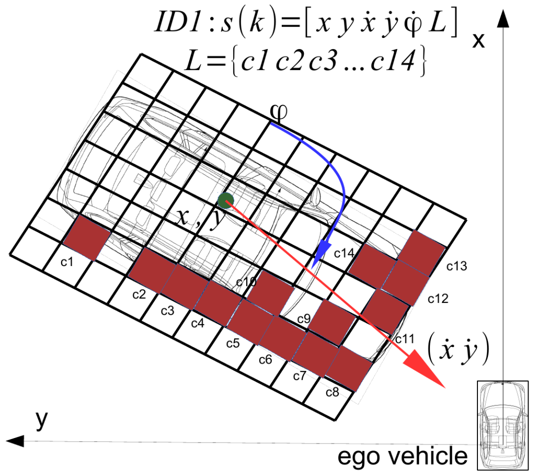

Figure 2: Object model in grid coordinates uses clustered dynamic cells (L). Grid map is rotated around ego vehicles course angle.

Algorithm 1: general particle filter algorithm see [13]

Data: previous particles $P_{f,t-1}$, control vector u_t, measurement z_t
Result: particle weights $w_k^{(i)}$, new particle set $P_{f,t}$

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1  P̄_{f,t} = P_{f,t} = 0 ;
2  for m = 1 ... M particles in P_{f,t-1} do
3      sample x_t^m with motion model p(x_t | u_t, x_{t-1}^m) ;
4      w_t^m = p(z_t | x_t^m) ;
5      // append calculated particle to set:
       P̄_{f,t} = P̄_{f,t} + {x_t^m, w_t^m} ;
6  end
7  for i = 1 ... M particles in P̄_{f,t} do
8      draw i with probability ∝ w_t^i ;
9      add x_t^i to P_{f,t} ;
10 end
11 return P_{f,t};
```

applied by drawing a sample in the state space with the probability $p(x_t|u_t, x^m_{t-1})$. After the prediction step the particle is weighted (line 4). The particle weight is determined by the likelihood of the agreement $p(z_t|x_t)$ between the particle state and current measurement (z_t). The object state of a particle filter is strongly modified by the weights of its particles (see 1 lines 8,9). Particle with low weights are replaced with high weighted particles. This is the typical survival of the fittest methodology of a particle filter.
As an advantage over other tracking algorithms, particle filters are able to track non-linear state spaces and noise can be modeled in any required form. We use particle filters as an experimental approach because they are easy to implement and offer the ability to track multiple object hypothesis in one filter.

3.2. Object Model

Taking advantage of the clustered dynamic cells proposed in the last section, particle filters are used to represent an object. Particles that track cells as proposed in [11] [12] have the property of converging on the measurement. This is no disadvantage as long as the whole object is visible in the majority of measurements, but it suffers when objects are partly occluded [14]. A box representation of the object to be tracked is used by [15] [16]. In an object box representation all measurements have to be fitted to a box. But a box might not be a good model for every situation. The state of a particle is proposed as $s(k) = (L, x, y, \dot{x}, \dot{y}, \omega)$ where $L = \{(c_1), (c_2), \ldots (c_n)\}$ is the set of clustered dynamic cells describing the associated measurement (see figure 2).
We use a Constant Turn Rate and Velocity (CTRV) model as state representation assuming the velocity $(\dot{x}, \dot{y})$ and the yaw rate ω are constant. The weight of a particle is estimated by the matching of the particle's object hypothesis and current measurements.

$$p(z^j_k|x^{(p)}_t) = \lambda \cdot |L_{z^j_k} \cap L_{x^{(p)}_t}| \tag{9}$$

The above equation is a matching function calculating the cluster (L) overlap of the object hypothesis of a particle $x^{(p)}_t$ and a measurement z^j_k. This weighting function could be described as sum over all dynamic beliefs of the current measurement grid cell, which matches with the cell cluster set L of the particle, normalized by the constant λ.
We combine all particles representing the same object in a particle set (P_f) in order to build a free form object model $\sum_{p \in P_f} L$ ($footprint$ in figure 3). Particles are initialized by a single dynamic cluster (L). A motion prediction step is performed in order to match the position of the particles to the new measurement. If all particles of a particle set are combined, they can be used as a free form model, namely $footprint$ F_p (see figure 3). The $fooprint$ combines all particles with its cell cluster L to an object representation in grid coordinates. The resulting object state is a set of cells described by a hit counter. This hit counter is incremented if the center of a tracked cell fits into an object local cell. Object local grids were proposed first by [17]. We modify this approach to accumulate a dynamic belief for a single object.

3.3. Track Management

The proposed tracking approach encompasses the following track management features:

- initialization of tracks using clustered dynamic cells

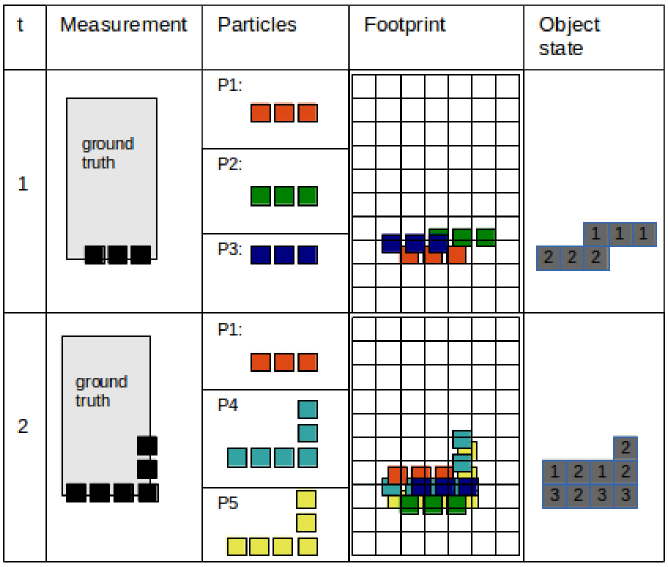


Figure 3: sequential build of free form object model using particles. In t=2 new particles p4 and p5 are inserted to enable the filter to track the new object shape.

- growing and shrinking of particle sets (i.e. adding or removing new cluster estimations to the set).
- deletion of divergent particle sets

Fusion of two tracks tracking the same object is beyond the scope of this paper.
A new particle set is set up if too few particles exist in the association region. To prevent false positives, the cell cluster used for initialization has to pass the constraint $\overline{B_D} \geq \overline{B_{\overline{D}}}$, where $(\overline{B_D})$ is the mean dynamic belief. As a first guess, all particles are initialized with the original dynamic cluster and a velocity in the interval $[V_s, -V_s]$. Only velocities within this interval can be tracked.

3.3.1. Growing

A particle filter once initialized by one dynamic cluster may describe multiple cluster representations. So new clusters should be inserted if the original shape of dynamic measurements does not match any more. Since we do not want to modify existing particles, we propose to add new particles to the particle filter. Therefore each particle filter has a maximum size $|P_{max}|$ but in the initial state all particle filters are able to grow or shrink. If a newly added cluster matches well it will replace other particles, which do not match anymore. That is a result of the survival of the fittest methodology of the used SIR particle filter (only the best particle are drawn). In [13] a similiar aproach for growing and shrinking of particle filters using $KLD - sampling$ algorithm is described.
A particle set should grow with a associated measurement that have more clustered cells $|C_{meas}|$ than the mean cluster size $|\overline{c}|$ in the particle set (see equation 10).

$$|P_f| = \begin{cases} |P_f| + \delta * (|P_{max}| - |P_f|), & if \beta \geq l_g \\ max(|P_{min}|, |P_f| - \gamma), & if |C_{Pos}| < l_s \end{cases} . \tag{10}$$

$$\delta = |\frac{|C_{meas}| - |\overline{c}|}{|\overline{c}|}|$$

Condition 1	Condition 2
$M \cap P_{fPos} = \emptyset$	$\lvert C_{Pos} \rvert \leq e$

Table 1: deletion condition

A new measurement should only be added to the set if it matches the track well $\beta \geq l_g$. Where β is the assignment probability that a track matches the measurement. This constraint avoids that wrong associations can break the object state.

3.3.2. Shrinking

In contrast to the growing process the shrinking process only needs to be called once for each particle filter. We define a minimal $|P_{min}|$ and maximal $|P_{max}|$ amount of particles for each particle set. $|C_{meas}|$ is the cluster size of the measured cluster associated with the track. Let $|\overline{c}|$ be the mean cluster size of the particle set P_f. The assignment probability β describes the likelihood that P_f is associated with C_{meas}. The shrinking of a particle set is triggered if the association probability is less than a certain threshold l_s. Additionally the growing of the particle filter is governed by the threshold l_g.
Recall that the positional covariance matrix C_{Pos} describes an ellipse which expresses the shape of the corresponding point cloud. Since shrinking is applied after all measurements have been associated, we have to compare against the eccentricity $|C_{Pos}|$. The amount of particles in a particle set ($|P_f|$) is changed adaptively with equation 10. The growing / shrinking function is applied at the weighting step of each particle filter.

3.3.3. Deletion

If any tracked object is not observable anymore, the particle set should be deleted. The deletion can be triggered by two conditions as listed in table 1. A particle filter should be deleted if the position of the tracked object P_{fPos} is not in the perception area of the map M. A second condition for deletion of an established track is passed if the filter has lost track of an object. This behaviour can be determined easily if the particle filter diverges.

3.4. Accumulating Dynamic Estimation for each Object

Validation of resulting tracked objects can be achieved by accumulating the dynamic belief of a measurement ($m_{S,k}(D)$) in a separate belief mass determined by the dynamic belief of an object. The belief mass $m_{O,k}(D)$ is stored as an object attribute. The dynamic belief for an object is updated using the associated measurement ($m_{S,k}$) of a cluster. We use the average of the dynamic beliefs of all relevant cells as the dynamic belief of a cluster.

$$m_{O,k}(D) = m_{O,k-1 \oplus S,k}(D) \tag{11}$$

$$m_{O,k}(\overline{D}) = m_{O,k-1 \oplus S,k}(\overline{D}) \tag{12}$$

Using Dempster's rule again to combine the measured mass with the prior dynamic estimation, a dynamic belief for each object can be maintained.

4. RESULTS

The proposed tracking algorithm and dynamic accumulation was tested on a vehicle equipped with two Ibeo LUX laser scanners. The environmental modeling using grid maps and the proposed tracking algorithm are illustrated for two different scenarios. Figure (4a, 4b, 4c) shows an urban environment with an ego velocity about 50 km/h. Figure (4d, 4e, 4f) shows a highway scenario with ego velocities about 160 km/h.
The Dempster Shafer belief masses are shown with the colours $green$ for free, red for occupied and $blue$ for the unknown belief mass (see figure 4c and 4f). The dynamic measurement ($yellow$) shown in figure 4b and 4e is strongly dependent on the estimated free space. With a higher belief of the free mass the estimation of the dynamic measurement is growing proportionally. This behaviour has the benefit that object close to the ego vehicle gain a higher dynamic belief. The colour $purple$ shows the belief of a not dynamic measurement.
The red cluster (in figure 4c and 4f) represents the tracked object $footprint$. The $yellow$ line describes the velocity vector of the vehicles. The $footprint$ of the overtaking vehicle in 4f consist of all shapes ever seen by one scanner. The initial shape of the object is only the side of the vehicle. Using the proposed model, the object shape grows with new measurements (if the back of the car is seen). The proposed particle filter is able to track any shape of an object.
In figure 4e the dynamic estimation of the right road boundary appear dynamic ($yellow$). This is caused by a misaligned mounting position of the right laser scanner during the test drive.
The algorithm was tested offline on an Intel Core i7 with 2.8 GHz. The map generation calculation time is about 3 ms. The tracking algorithm including clustering and some debug outputs is about 10 ms.

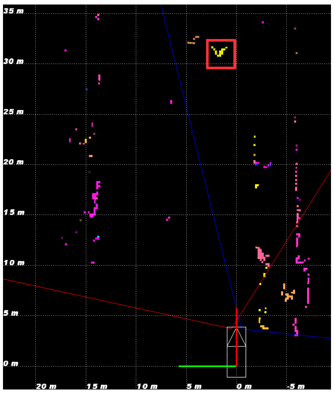
4.1. Implementation Details

The cell size of the used occupancy grid is 20 cm and we built the occupancy grid of size of 102.4 m x 102.4 m. We statically allocate memory for all particles ever used in the algorithm up to a maximum of 8000 particles. Each particle filter starts with 375 particles ($|P_{min}|$) and is able to grow to a size of 500 particles ($|P_{max}|$).

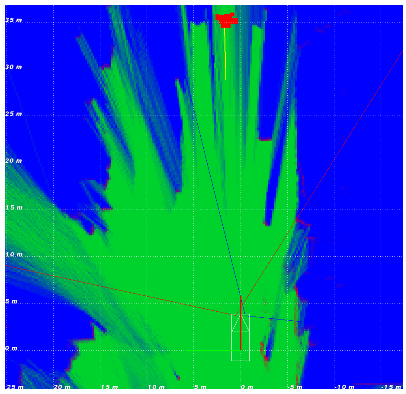
5. CONCLUSION AND FUTURE WORK

In this paper an environmental modeling technique for vehicles equipped with multiple laser scanners is presented. We propose to build a layered occupancy grid map to extend the model of a static environment with information produced by dynamic objects. These dynamic information could be clustered and tracked using particle filters. As free form object model, multiple particles generate an object local grid map. As mentioned in [5] the two types of cell conflict C_1 and C_2 could be used for estimating the direction of the motion. A direction could be useful for a faster track initialization of the used particle filter algorithm. The initial state of each particle could be initialized by a first estimation of the direction in order to improve the convergence speed of the particle filter.
We plan to compare the implemented particle filter tracking against a Kalman filter tracking comparing initialization time and position error.

(a) video view as reference for the urban scenario

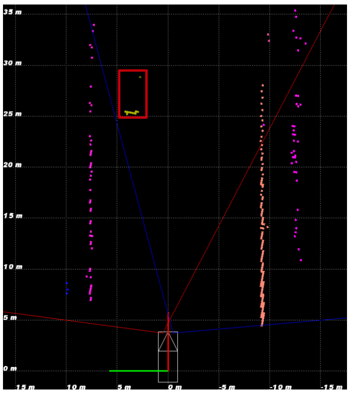


(b) dynamic measurement grid map

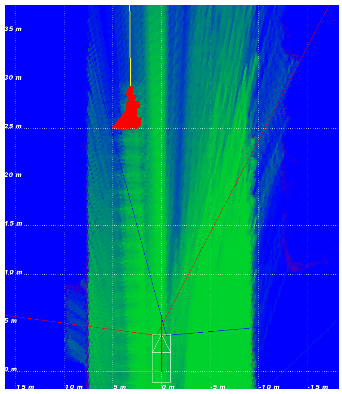


(c) occupancy grid map with tracked object as footprint (red cell cluster)

(d) video view as reference for the highway scenario



(e) dynamic measurement grid map



(f) occupancy grid map with tracked object as footprint (red cell cluster)

Figure 4: Grid based tracking and dynamic detection in an urban and highway scenario. The dynamic measurement grid shows yellow for the dynamic belief mass and purple for the not dynamic belief mass.

6. ACKNOWLEDGEMENTS

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